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Math Bite: Axial View of Trigonometric Functions

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A typical way to picture the sine and cosine functions is shown in FIGURE 1, where a given central angle θ appears in its standard position in the unit circle of the Cartesian plane. Since the horizontal distance OM is $\cos\theta$, we may loosely call the horizontal axis the *cosine axis*. Similarly, the vertical distance OL is $\sin\theta$ and so the the sine function is associated with the vertical axis. An advantage of this approach is that students can reliably determine the correct signs for these ratios when θ is outside the first quadrant. You may be familiar with similar axes associated with the tangent and cotangent functions, but have you ever thought of a secant axis?

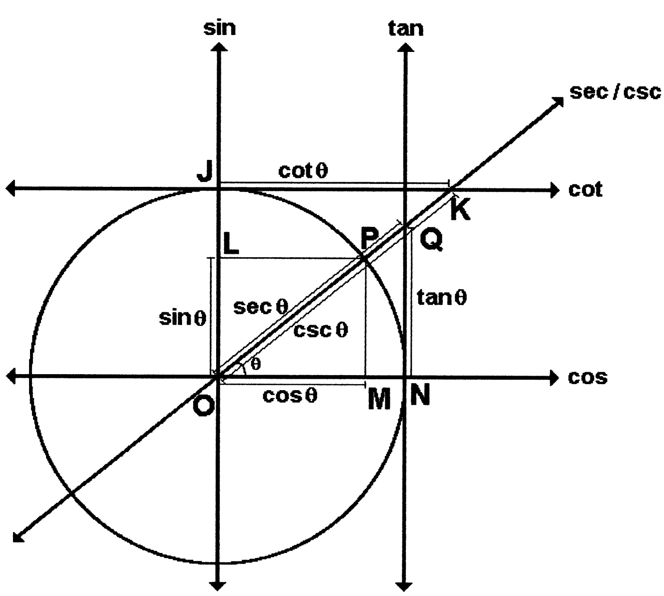

Figure 1 An angle in standard position in the first quadrant

Consider the directed lines perpendicular to the coordinate axes at N(1,0) and J(0,1), oriented in the same way as the axes to which they are parallel. Since $\tan\theta = QN$ and $\cot\theta = JK$, these axes could be called the tangent axis and cotangent axis. Positive and negative values of the tangent and cotangent functions will be determined correctly when read from these axes, as you can check in FIGURE 2.

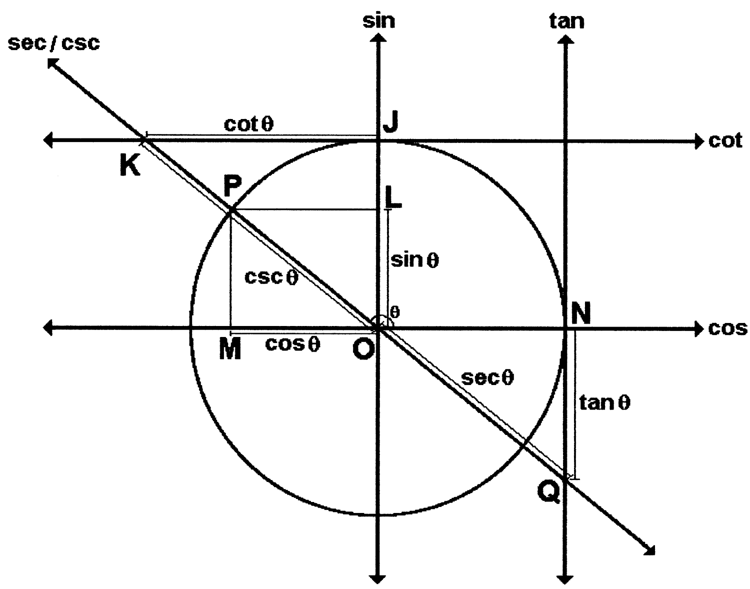

Figure 2 The second quadrant

To represent the secant and cosecant functions, we use the directed line that forms the terminal side of θ. While this axis does move as θ changes, the values of the secant and cosecant functions can be read off as lengths along it. In particular, $\sec\theta = OQ$ and $\csc\theta = OK$. Thus, the intersection of this sec/csc axis with the tangent axis determines the secant, and its intersection with the cotangent axis determines the cosecant of the angle. As you can see, in the first quadrant $\sec\theta$ and $\csc\theta$ are both positive, since they are both measurements along the positive side of sec/csc axis. In the third quadrant, the intersection of sec/csc axis with the tan and cot axes both occur along the negative side of the sec/csc axis and so $\sec\theta$ and $\csc\theta$ are both negative.

In the second quadrant (FIGURE 2), the intersection of the sec/csc axis with the tangent axis occurs along the negative side of the sec/csc axis and the intersection with the cotangent axis occurs along its positive side. As a result, $\sec\theta$ is negative and $\csc\theta$ positive in this quadrant. Finally, in the fourth quadrant, the intersection of the sec/csc axis with the tangent axis occurs along the positive side of the sec/csc axis, and with the cotangent axis along its negative side. Thus, sec θ is positive and csc θ is negative in this quadrant.

The axial view delivers the correct answer for exceptional angles where some of the trigonometric functions are undefined: the $\sec(\pi/2)$ does not exist, and the sec/csc axis does not intersect the tangent axis. Overall, this visual interpretation should help students develop an intuitive sense of the magnitudes and signs of trigonometric functions.

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